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On the stabilization of a two-dimensional vortex strip by adverse shear

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An isolated strip of anomalous vorticity in a two-dimensional, inviscid, incompressible, unbounded fluid is linearly unstable – or is it? It is pointed out that an imposed uniform shear, opposing the shear due to the isolated strip alone, can prevent all linear instabilities if the imposed shear is of sufficient strength, and that this is highly relevant to current thinking about 'two-dimensional turbulence' and related problems. The linear stability result has been known and goes back to Rayleigh, but its implications for the behaviour of the thin strips of vorticity that are a ubiquitous feature of nonlinear two-dimensional flows, as revealed for instance in high-resolution experiments, appear not to have been widely recognized. In particular, these thin strips, or filaments, almost always behave quasi-passively when being wrapped around intense coherent vortices, and do not roll up into strings of miniature vortices as would an isolated strip. Nonlinear calculations presented herein furthermore show that substantially less adverse shear than suggested by linear theory is required to preserve a strip of vorticity. Taken together, and in conjunction with results showing the further stabilizing effect of a large-scale strain field, these results explain the observed quasi-passive behaviour.

1. Introduction

Recent very high-resolution laboratory and numerical experiments of two-dimensional flow (Couder & Basdevant 1986; Benzi, Patarnello & Santangello 1987; Brachet *et al.* 1987; Dritschel 1988*a*, *b*, *c*; Juckes & McIntyre 1987; Legras, Santangello & Benzi 1988; Melander, McWilliams & Zabusky 1987*a*; Melander, Zabusky & McWilliams 1987*b*, *c*) have demonstrated more clearly than ever before the formation and long-time persistence of thin strips, or filaments, of anomalous vorticity. Such strips or filaments are apparently a ubiquitous feature of very high Reynolds number, unsteady two-dimensional vortical flow, and of analogous three-dimensional stably stratified flows that are commonplace in the Earth's atmosphere and other naturally occurring bodies of fluid.

The persistence of these strips might be thought to contradict the familiar classical linear stability result (Rayleigh 1894, 1945) that demonstrates the instability of an isolated strip of uniform vorticity. Furthermore, experiments and calculations of the nonlinear evolution of a shear layer (a strip of anomalous vorticity) show that the layer develops into a string of vortices that subsequently pair repeatedly (Thorpe 1968; Winant & Browand 1974; Saffman & Baker 1979; Aref & Siggia 1980; Pierrehumbert & Widnall 1981; Aref 1983; Ho & Huerre 1984; Pozrikidis & Higdon 1985; Pullin & Jacobs 1986, among others.) This behaviour, however, is almost never observed in the experiments first cited.

In those experiments, thin strips of vorticity are almost always created when coherent vortices strongly interact. Strong interactions include vortex merging (Melander *et al.* 1987*b*, *c*; Dritschel 1988*a*), 'binding' of opposite-signed vorticity (Overman & Zabusky 1982; Couder & Basdevant 1986), 'axisymmetrization' of sufficiently non-circular distributions of vorticity (Melander *et al.* 1987*a*; Dritschel 1988*a*; Dritschel & Legras 1989), the deformation and elongation of small vortices by the strain field of larger, more intense vortices (Moore & Saffman 1971; Kida 1981; Neu 1984; Dritschel 1989), and unsteady external forcing (see e.g. the stratospheric model of Juckes & McIntyre 1987). In many circumstances, strips then thin under the action of the strain arising from the presence of a nearby vortex, because the part of a strip closest to the vortex is swept around the vortex more rapidly than the parts further out (e.g. see figure 1).

Thus, one possible explanation for the persistence of thin strips could be that the strain field stretches the strip fast enough to prevent the growth of instabilities. In a companion paper, Dritschel *et al.* (1989), it is shown that a weak uniform strain, whose extensional axis lies along the centreline of an undisturbed strip of uniform vorticity, can indeed suppress instabilities (see also Dhanak 1981 & references). A strain rate of only 7% of the vorticity maximum is sufficient to keep the steepness of a disturbance from growing by more than a factor of e.

In general, however, the strain field is neither constant in time, nor does the extensional axis of strain remain precisely parallel with the strips of vorticity. As a strip of vorticity stretches under the action of differential rotation about an intense vortex, the strip also becomes less and less aligned with the extensional axis of strain (e.g. see figure 1). The rate of stretching by strain thereby diminishes continually, and eventually the stretching is no longer sufficient to prevent instabilities, at least by the mechanism discussed above (for quantitative results, see Dritschel *et al.* 1989).

Although the stretching effect diminishes, thin strips of vorticity continue to be strongly influenced by differential rotation about the vortex. In itself, this tends to make the inner 'edge' of the strip rotate faster than the outer, because the circumferential velocity decreases with increasing distance from the vortex. This shearing motion induced by the main vortex opposes the self-induced shearing motion of the strip, so that the net shear across the strip is reduced or even reversed.

The purpose of this paper is to show that this adverse shear is likely to be the main factor responsible for the observed persistence or stability of thin strips of vorticity being wrapped around intense coherent vortices. The following section reviews the classical linear stability results of Rayleigh (1945, vol. 2, p. 388) without and with adverse shear, and takes them further by examining the effect of weak adverse shear on the strength and nature of the instabilities. In §3, the sensitivity of the stability properties to the distribution of vorticity across the strip is addressed. Thin circular strips are considered in §4 just to make sure that geometrical effects do not alter the results of the previous two sections significantly. In §5, the linear stability results are connected with the long-established 'inflection-point' theorems of Rayleigh (1894), Fjortoft (1950), and Arnol'd (1965). In §6, the nonlinear evolution of small, linearly unstable disturbances is examined. The evolution is shown to fall into several qualitatively different regimes depending on the value of the adverse shear. In particular, an adverse shear of between about two-thirds and unity times the vorticity within the strip can prevent disruption of the strip even though it does not suppress the primary linear instability (on the assumption that subharmonics are unimportant). The most unstable linear disturbance amplifies at first but then returns, repeatedly, close to its initial amplitude. Remarkably, wave breaking and filamentation appear to be completely, or almost completely, suppressed.

A discussion, including wider implications, is given in §7.

2. The stability of a strip of uniform vorticity

The classic problems solved by Rayleigh are now described. In the case of no imposed adverse shear, the equilibrium configuration is defined as a region of uniform vorticity ω bounded by the two lines $y = \pm\frac{1}{2}\Delta$ beyond which the fluid is irrotational. (Throughout this paper, the fluid is assumed inviscid, incompressible, two-dimensional, unforced, and unbounded.) The strip is viewed in a reference frame in which the centreline position of the strip is motionless, and only the component $\bar{u}$ of the velocity $(\bar{u}, \bar{v})$ parallel to the x-axis (the axis of the strip) is non-zero; $\bar{u} = \mp\frac{1}{2}\omega\Delta$ above and below the strip respectively, and within the strip, $\bar{u} = -\omega y$.

The linear stability of this equilibrium is obtained by adding small, normal-mode disturbances

$$\eta_{\pm}(x,t) = \mathrm{Re}\,[\hat{\eta}_{\pm}\exp(\mathrm{i}kx - \mathrm{i}\sigma t)] \quad (k > 0)$$

to the upper and lower boundaries of the strip, linearizing the equations of motion (the Euler equations), and forcing continuity of the perturbation cross-strip velocity v and pressure p at each of the boundaries. Equivalently (and more simply), one can begin directly from the equations of motion expressed in terms of contour integrals along the two interfaces of vorticity discontinuity (Zabusky, Hughes & Roberts 1979). In either case, one obtains two coupled equations for the complex amplitudes $\hat{\eta}_{\pm}$,

$$\left(\frac{\sigma}{\omega} - \tfrac{1}{2}(1-k\Delta)\right)\hat{\eta}_{+} + \tfrac{1}{2}\mathrm{e}^{-k\Delta}\hat{\eta}_{-} = 0, \tag{1}$$

$$\left(\frac{\sigma}{\omega} + \tfrac{1}{2}(1-k\Delta)\right)\hat{\eta}_{-} - \tfrac{1}{2}\mathrm{e}^{-k\Delta}\hat{\eta}_{+} = 0, \tag{2}$$

whose solution requires that the eigenvalue σ take the value

$$\sigma(k) = \pm\tfrac{1}{2}\omega[(1-k\Delta)^2 - \mathrm{e}^{-2k\Delta}]^{\frac{1}{2}}. \tag{3}$$

Disturbances with non-dimensional wavenumbers $k\Delta$ between 0 and 1.278 464 54 ... are unstable, and the most unstable mode occurs for $k\Delta = 0.796\,812\,13\ldots$ for which $\mathrm{Im}\,(\sigma) = 0.201\,185\,58\ldots\omega$.

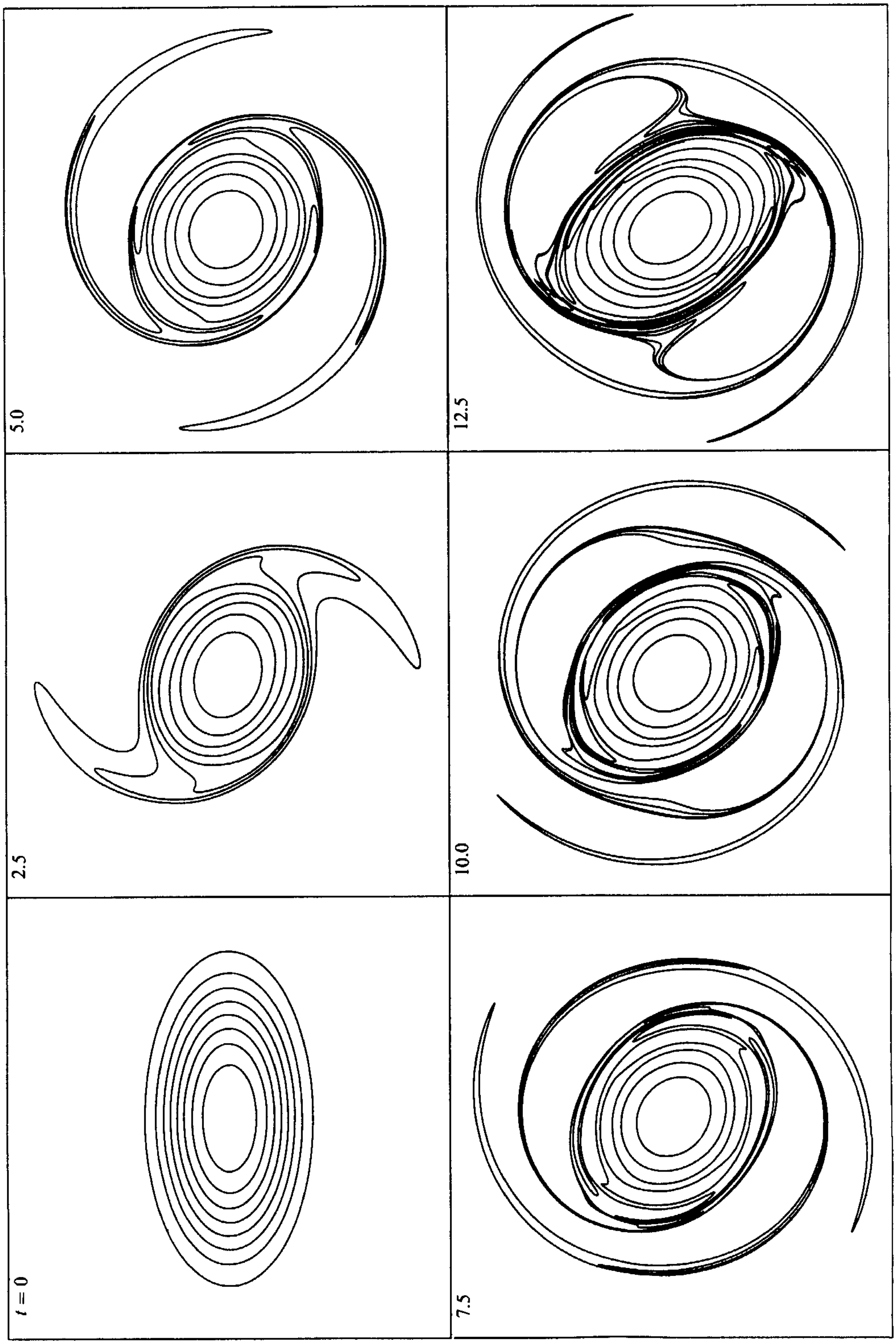
Consider next the effect of adverse shear. The equilibrium flow in this case departs from that above simply by the addition of uniform shear, $\omega\Lambda y$, everywhere and parallel to the axis of the strip. Note, in particular, that the equilibrium velocity $\bar{u}$ within the strip now assumes the form $\bar{u} = -\omega(1-\Lambda)\,y$. Linear stability is determined in precisely the same fashion as above. The complex amplitudes $\hat{\eta}_{\pm}$ satisfy

$$\left(\frac{\sigma}{\omega} - \tfrac{1}{2}(1-k\Delta(1-\Lambda))\right)\hat{\eta}_{+} + \tfrac{1}{2}\mathrm{e}^{-k\Delta}\hat{\eta}_{-} = 0, \tag{4}$$

$$\left(\frac{\sigma}{\omega} + \tfrac{1}{2}(1-k\Delta(1-\Lambda))\right)\hat{\eta}_{-} - \tfrac{1}{2}\mathrm{e}^{-k\Delta}\hat{\eta}_{+} = 0, \tag{5}$$

whose solution now requires that the eigenvalue σ takes the value

$$\sigma(k,\Lambda) = \pm\tfrac{1}{2}\omega[(1-k\Delta(1-\Lambda))^2 - \mathrm{e}^{-2k\Delta}]^{\frac{1}{2}}. \tag{6}$$


t = 0
2.5
5.0
7.5
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12.5

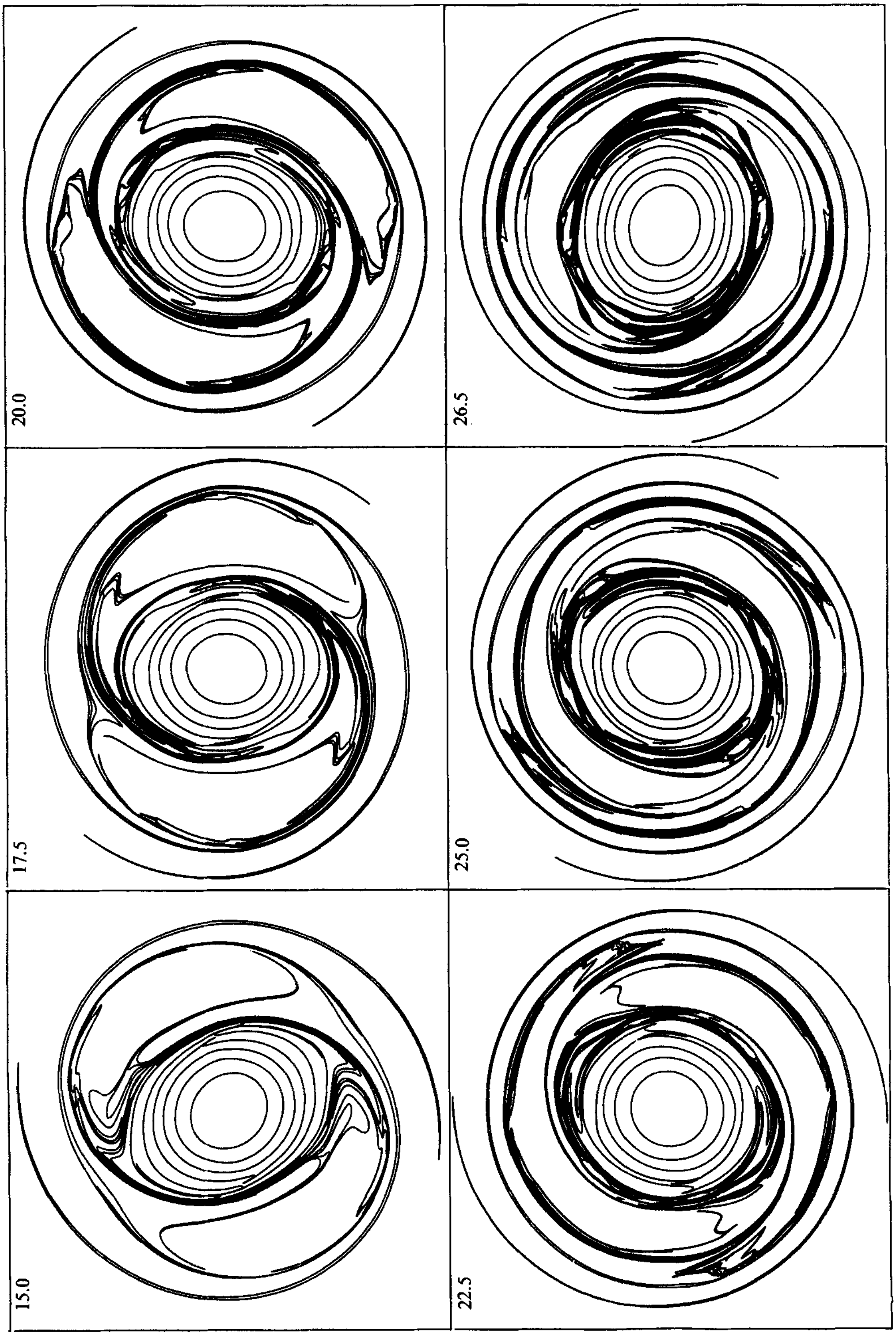


FIGURE 1. For caption see page 198.

One can immediately conclude that there is stability if $\Lambda \geqslant 1$, i.e. if the adverse shear exceeds the vorticity jump crossing into the strip, because then the quantity within the radical in (6) cannot be negative for any value of the wavenumber k. Physically, values of $\Lambda \geqslant 1$ correspond to a reversal of the sign of the mean velocities, $\bar{u}$, on the two interfaces, and this reversal makes it impossible for waves of any wavelength to stay in step with one another (see §6b of Hoskins, McIntyre & Robertson 1987).

More details of the dispersion relation (6) are discussed in the following section, so only a brief examination is given here. When Λ is very nearly unity, say $\Lambda = 1-\epsilon$, $\epsilon \ll 1$, the mode which is most unstable has $k\Delta \approx 1/\epsilon$, corresponding to a small wavelength, and $\mathrm{Im}\,(\sigma) \approx \frac{1}{2}\omega\,\mathrm{e}^{-1/\epsilon}$, corresponding to a vanishingly small growth rate, and the range of unstable wavenumbers shrinks to $1-\mathrm{e}^{-1/\epsilon} \leqslant \epsilon k\Delta \leqslant 1+\mathrm{e}^{-1/\epsilon}$, approximately. That is, when the net shear within the interface is very weak, only waves with small wavelengths, waves that propagate at very nearly the mean flow speed at the two edges of the strip, can stay in step with one another and therefore cause instability. But because the waves have such small wavelengths, their mutual interaction is extremely weak (like $\mathrm{e}^{-k\Delta}$ for $k\Delta \gg 1$), and therefore the growth rates are practically negligible.

3. More general cross-strip distributions of vorticity

How do the results of the previous section carry over to strips whose vorticity distributions are not uniform? In this section, we consider the linear stability of a particular class of distributions and afterwards hypothesize the linear stability of other classes.

In the absence of adverse shear, the equilibrium vorticity profile $\bar{q}(y)$ is taken to be uniform and equal to ω for $|y| \leqslant a\Delta$ and Gaussian,

$$\omega \exp\left(-(|y|-a\Delta)^2/\epsilon^2\right),$$

for $|y| > a\Delta$. The constant a is chosen so that the circulation within the strip is the same as that for the uniform-vorticity strip considered in the previous section, and this requires $a = \frac{1}{2}(1-\pi^{\frac{1}{2}}\epsilon)$. The distribution is entirely Gaussian when $\epsilon = 1/\pi^{\frac{1}{2}} = 0.564189\ldots$, while it is entirely uniform when $\epsilon = 0$.

Actually, this distribution of vorticity is modified slightly in order to simplify the stability problem. The continuous distribution is replaced by $m = 32$ discrete steps symmetrically located to either side of the strip's centre. The vorticity jumps by an equal amount, ω/m, across each of these steps. Half of the steps lie, in equilibrium, at the positions

$$y_j = \left[a+\epsilon\left(-\log\left(\frac{m+1-j}{m}\right)\right)^{\frac{1}{2}}\right]\Delta \quad (j = 1, 2, \ldots, m), \tag{7}$$

FIGURE 1. A contour-surgery calculation of a free vortex having an initially elliptical distribution of vorticity (see Dritschel & Legras 1989). The vorticity distribution consists of 8 equal steps with vorticity increasing inwards to a value of 2π at the centre. Time advances to the right and downwards. The vortex initially ejects two broad filaments of vorticity, which subsequently thin as a result of the straining action caused by the differential rotation about the core of the vortex. The filaments become progressively more aligned with the predominantly circular flow field, and resist rolling up by virtue of the flow about the core. Where and when one does see enlargements along the filaments, the cause is not the classical Rayleigh instability, but the reaction to the strong differential rotation carrying around the parts of filaments close to the vortex faster than the parts further out; otherwise, the cause is related to the slight but persistent elliptical shape of the interior core.

and the other half lie at the symmetrical positions $-y_j$. The associated equilibrium along-strip velocity profile $\bar{u}(y)$ is obtained by integrating the (negative of the) vorticity across the strip. Thus, in a frame of reference for which the centre of the strip is stationary, the velocity $\bar{u}$ at $y = y_j$, $\bar{u}_j$, is given by the recursive formula

$$\bar{u}_j = \bar{u}_{j-1} - \left(\frac{m+1-j}{m}\right)(y_j - y_{j-1})\,\omega \quad (\bar{u}_0 = y_0 = 0). \tag{8}$$

$\bar{u}$ at $y = -y_j$ is simply the opposite of $\bar{u}_j$.

To include adverse shear, it is only necessary to augment $\bar{u}_j$ by $\omega \Lambda y_j$.

Linear stability is determined by a way precisely analogous to that outlined in the previous section. To the position of each step $\pm y_j$, we add a disturbance of the form

$$\eta_{\pm j}(x,t) = \mathrm{Re}\,[\hat{\eta}_{\pm j} \exp(\mathrm{i}kx - \mathrm{i}\sigma t)] \quad (k > 0).$$

Upon linearizing the equations of motion, one obtains a set of $2m$ coupled equations for the complex amplitudes $\hat{\eta}_{\pm j}$

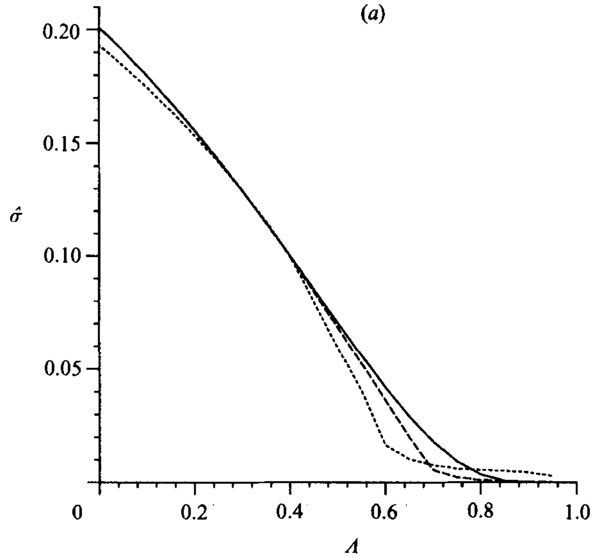
$$(\sigma \mp k\bar{u}_j)\,\hat{\eta}_{\pm j} = \frac{\omega}{2m} \sum_{l=1}^{m} \hat{\eta}_{+l} \exp(-k|y_l \mp y_j|) - \hat{\eta}_{-l} \exp(-k|y_l \pm y_j|) \tag{9}$$

whose solvability hinges on σ being determined as an eigenvalue. In general, σ could take any one of as many as $2m$ distinct values corresponding to $2m$ distinct modes. In the following, though, attention is restricted only to the mode with the largest growth rate.

A comparison of linear stability characteristics is next made between three distributions of vorticity, differentiated by the value of ϵ which gives a measure of the reciprocal steepness of the vorticity gradients. The first distribution has $\epsilon = 0$ – this is just the uniform-vorticity case discussed in the previous section. The second and third distributions have $\epsilon = 0.2$ and 0.5 respectively, the latter distribution, being nearly Gaussian, is the least steep.

The stability characteristics of the three distributions are compared on the basis of the maximum growth rates for a given value of non-dimensional adverse shear Λ, and the corresponding dimensionless wavenumber $k_{\mathrm{m}} \Delta$ which gives rise to the most unstable mode. The maximum growth rate as a function of Λ for the three distributions is plotted in figure 2(*a*). All of the distributions are stable for $\Lambda \geqslant 1$. What is most surprising though is that the three curves are very similar. The small discrepancies near $\Lambda = 1$ arise because, unlike in the case of the uniform-vorticity distribution, non-uniform distributions allow for internal modes of instability. This is most easily seen in figure 2(*b*) which plots $k_{\mathrm{m}} \Delta$ versus Λ for the three distributions. The rough variation of the curves for $\epsilon = 0.2$ and 0.5 as Λ approaches unity reflects a competition between various modes of instability having different internal structures.

The results of figure 2(*a*) suggest a certain ubiquity to the pattern of growth rate versus adverse shear. Similar stability results are therefore hypothesized for all other 'one-humped' distributions of vorticity. The fact that stability always results when the adverse shear equals or exceeds the maximum vorticity anomaly in the flow is explained below in §5.



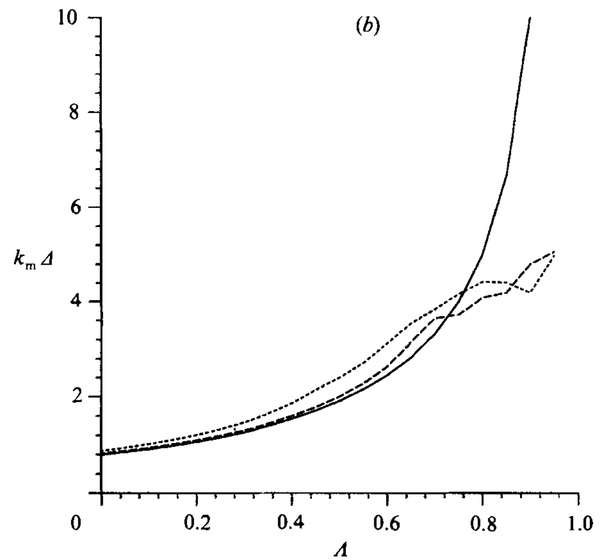


FIGURE 2. (a) The maximum dimensionless growth rate $\hat{\sigma} = \mathrm{Im}\,(\sigma/\omega)$ versus the dimensionless adverse shear Λ for three cross-strip profiles of vorticity: $\epsilon = 0$ (solid line), $\epsilon = 0.2$ (dashed line), and $\epsilon = 0.5$ (dotted line). (b) The dimensionless wavenumber $k_m \Lambda$ of maximum instability versus the dimensionless adverse shear Λ for three cross-strip profiles of vorticity (same line coding as in a).

4. Circular strips of vorticity

The strips of vorticity which are produced from vortex interactions in general wind around intense coherent structures, and so retain a curved shape. Do the results for straight strips continue to apply in this case? This question is addressed here by examining the stability of a thin circular strip with uniform vorticity in the presence of adverse shear.

Consider the basic state defined by a ring of uniform vorticity ω bounded inside by the circle $r = a$ and outside by the concentric circle $r = b$. Adverse shear is introduced by situating a *point* vortex of circulation Γ at the origin. The point vortex is meant to represent an intense coherent vortex lying within the ring. The question as to whether we are missing out on some possible instabilities by not giving the central vortex internal degrees of freedom is discussed further below.

A uniformly rotating frame of reference is chosen so that the two edges of the strip, in equilibrium, rotate at the same rate but in opposite directions. Then, the outer edge rotates at the rate $\Omega = \frac{1}{4}\omega(1-\tilde{a}^2)(1-\Lambda)$ where $\tilde{a} = a/b$ and

$$\Lambda = \frac{\Gamma}{\pi a^2 \omega} \tag{10}$$

is the dimensionless 'adverse shear'. Thus, a sufficiently strong central vortex can overcome the shear due to the strip alone (the vortex can reverse the angular velocities on the inner and outer edges of the strip).

Linear stability is determined by adding disturbances of the form

$$\eta_{\pm}(\theta, t) = \mathrm{Re}\,[\hat{\eta}_{\pm} \exp(\mathrm{i}m\theta - \mathrm{i}\sigma t)] \quad (m = 1, 2, 3, \ldots) \tag{11}$$

to the outer and inner edges of the ring, respectively, linearizing the equations of motion, and solving a two-by-two determinant for the eigenvalue σ. The result is

$$\sigma = \pm\tfrac{1}{2}\omega\{[1-\tfrac{1}{2}m(1-\tilde{a}^2)(1-\Lambda)]^2 - \tilde{a}^{2m}\}^{\frac{1}{2}}. \tag{12}$$

It is easy to see that $\Lambda \geqslant 1$ stabilizes the ring regardless of the value of $\tilde{a}\,(0 < \tilde{a} < 1)$. And, in the limit of a thin ring, $\Delta = b - a \ll a$, one obtains precisely the same dispersion relation as governs a straight strip of vorticity, with the identification $k = m/a$. The stability of a thin circular strip is not therefore essentially different from that of a straight strip.

A point vortex stabilizes a circular strip of vorticity by reversing the angular velocity shear across the strip. Analogously, adverse shear stabilizes a straight strip by reversing the linear velocity shear across the strip. The fact that the stabilizing shear flow is irrotational in one situation but rotational in the other has no affect on the stability results.

Consider next replacing the central point vortex by a finite-area vortex of uniform vorticity ω_0. In equilibrium, the edge of the vortex is chosen to lie at $r = R < a < b$, a and b being the inner and outer edges of the strip as before. Unlike the point vortex, a finite-area vortex may now change its shape in response to the strip, and there arises the possibility of additional instabilities.

The linear stability of this flow is most easily discussed in terms of the general equation governing the stability of all circularly symmetric flows with piecewise-constant vorticity. Let r_j, $j = 1, 2, \ldots$ denote the equilibrium radial positions of the boundaries of vorticity discontinuity, with $r_1 < r_2 < \ldots$, and let $\tilde{\omega}_j$ denote the jump in vorticity crossing $r = r_j$ inwards. Then the stability of small disturbances of the form

$$\eta_j(\theta, t) = \mathrm{Re}\,[\hat{\eta}_j \exp(\mathrm{i}m\theta - \mathrm{i}\sigma t)] \quad (m = 1, 2, 3, \ldots) \tag{13}$$

is determined by solving the following eigenvalue problem:

$$(\sigma - m\Omega_j)\,\hat{\eta}_j + \tfrac{1}{2}\sum_l \tilde{\omega}_l I_{mlj}\,\hat{\eta}_l = 0, \tag{14}$$

where Ω_j denotes the equilibrium angular velocity at $r = r_j$, and $I_{mlj} = (r_j/r_l)^{m-1}$ when $j < l$ and $I_{mlj} = (r_l/r_j)^{m+1}$ when $j \geqslant l$.

For the example flow above, $r_1 = R$, $r_2 = a$ and $r_3 = b$ while $\tilde{\omega}_1 = \omega_0$, $\tilde{\omega}_2 = -\omega$ and $\tilde{\omega}_3 = \omega$. A uniformly rotating frame of reference is chosen so that $\Omega_3 = -\Omega_2 = \Omega =$

$\frac{1}{4}\omega(1-(a/b)^2)(1-\Lambda)$ with $\Lambda = \omega_0 R^2/\omega a^2$ (cf. (10) in which $\Gamma = \pi\omega_0 R^2$). In this frame, $\Omega_1 = \frac{1}{2}(\omega_0 - \omega\Lambda) - \Omega$. The eigenvalue σ satisfies a cubic equation depending essentially upon the four parameters m, a/b, R/a, and ω_0/ω, and no attempt is made to discuss the full spectrum of behaviour in this four-dimensional space. Instead, attention is restricted to the thin-strip limit, $\Delta = b - a \ll a - R$. Upon introducing the dimensionless quantities

$$\hat{\sigma} = \frac{\sigma}{\frac{1}{2}\omega}, \quad \kappa = \frac{m\Delta}{a}, \quad \beta = \kappa(1-\Lambda), \quad \hat{\omega} = \frac{\omega_0}{\omega},$$

$$\hat{\Omega} = m\hat{\omega}\left(1 - \left(\frac{R}{a}\right)^2 - \frac{1}{m}\right) - \beta, \quad \alpha = \mathrm{e}^{-2\kappa}, \quad \gamma = \hat{\omega}\left(\frac{R}{a}\right)^{2m}$$

the cubic equation may be written

$$(\hat{\sigma} - \hat{\Omega})(\hat{\sigma}^2 - (1-\beta)^2 + \alpha) + \gamma((1-\alpha)(\hat{\sigma}+1) - (1+\alpha)\beta) = 0. \tag{15}$$

Two limits of (15) are examined in detail. In the first, $\kappa (= m\Delta/a) = O(1)$ so that $m \gg 1$. In this limit, $\gamma = \Lambda(R/a)^{2(m-1)}$ is negligible by virtue of the fact that $\Delta \ll a - R$, assuming Λ is not large, and one obtains the same stability results as in the case of a straight strip in uniform shear (the first example of the previous section). A third, neutral mode ($\hat{\sigma} = \hat{\Omega}$) is present describing the rapid propagation of waves on the boundary of the central vortex.

In the second limit considered, $\kappa \ll 1$ so that m is not large and therefore γ cannot be neglected. Curious behaviour is found to occur for small $\hat{\Omega}$, when $(R/a)^2 \approx 1-(1/m)$. Setting $\hat{\Omega} = c\kappa^{\frac{1}{3}}$, one can prove that $c < 3(\gamma\Lambda/2)^{\frac{1}{3}}$ results in instability, with maximal instability occurring at $c = 0$ to the tune of $\mathrm{Im}(\sigma) = \frac{1}{4}\sqrt{3}\omega(2\gamma\Lambda\kappa)^{\frac{1}{3}}$. Thus, for a given size of the central vortex, there may exist instabilities, involving the amplification of disturbances of approximately equal magnitude on the strip and on the central vortex, growing at a rate proportional to $\omega\Lambda^{\frac{2}{3}}(\Delta/a)^{\frac{1}{3}}$. The dependence of σ on the *third* root of the non-dimensional strip thickness, Δ/a, implies, in practical situations, that even an extremely thin strip can excite sizable growth rates, at least relative to the strip's own vorticity (a nonlinear calculation of this instability is given in §6). However, relative to the typically much greater vorticity within the central vortex, these growth rates are small.

5. Sufficient conditions of linear stability

Following Rayleigh (1894), Fjortoft (1950) derived the following sufficient condition of the stability of parallel flow. Linear stability is assured so long as there exists a uniformly translating frame of reference such that the zonal velocity $\bar{u}(y)$ satisfies

$$\bar{u}\bar{u}_{yy} = -\bar{u}\bar{q}_y \geqslant 0 \tag{16}$$

everyhwere. For piecewise-constant vorticity, $\bar{q}_y$ is a series of delta functions, $-\tilde{\omega}_j \delta(y-y_j), j = 1, 2, \ldots$, but (16) still implies stability as long as $\bar{u}_j \tilde{\omega}_j \geqslant 0$ for all j.

This sufficient condition for stability is in fact satisfied for the parallel basic flows considered in §§2 and 3 when the dimensionless adverse shear $\Lambda \geqslant 1$, i.e. when the ratio of the adverse shear to the peak vorticity anomaly is at least unity.

Arnol'd (1965) further generalized Rayleigh's and Fjortoft's results to include wide classes of non-parallel flows. For circularly symmetric basic flows, the sufficient condition for linear stability is that the angular velocity $\bar{\Omega}(r)$, in *some* uniformly rotating reference frame, must satisfy

$$\bar{\Omega}\bar{q}_r \geqslant 0 \tag{17}$$

everywhere. For piecewise-constant vorticity, this condition becomes $\tilde{\Omega}_j \tilde{\omega}_j < 0$ for all discontinuities j.

This condition explains why a sufficiently strong point vortex can stabilize a circular strip of vorticity: the differential rotation arising from the point vortex overcomes the strip's own opposing differential rotation, so that the inner edge of the strip rotates faster than the outer. But note that the condition for stability is *never* satisfied when the strip surrounds a finite-area vortex, the case considered in the second part of the previous section. Although this does not imply instability, the case considered demonstrates that instability can indeed occur.

6. Nonlinearity

In this section, direct numerical calculations are used to examine the nonlinear development of the linearly unstable flows considered in §§2–4. The calculations were performed using a numerical technique specifically designed for piecewise-constant vorticity called 'contour surgery' (see Dritschel 1988*a* for details regarding basic parameters, e.g. the time step, and accuracy). The technique is a variant and refinement of the contour-dynamics methods pioneered by Zabusky *et al.* (1979).

In the case of a straight, uniform strip of vorticity, a series of calculations was performed to determine the nonlinear development of initially small, unstable disturbances for different values of the ratio Λ of adverse shear to vorticity. Each calculation was performed in a periodic domain of length 2π and began with the most unstable eigenmode for a given value of Λ. The wavenumber of the disturbance k always equals unity, the width of the undisturbed strip $\Delta(\Lambda)$ is chosen from figure 2(*b*) ($k_m = 1$), and the amplitude of the disturbance (on both interfaces) is chosen to be 0.05Δ.

In all of the calculations, accuracy is gauged by measuring the accumulative error in the conservation of area (circulation) over the course of each calculation (see table 1). A sense of the passage of time in a calculation can be appreciated by noting that the vorticity in the strip takes the uniform value 2π. The numerical algorithm parameters (see Dritschel 1988*a*) are $\Delta t = 0.05$, $\mu = 0.04$, and $\delta = \frac{1}{8}\mu^2 = 0.0002$, except in one case when $\mu = 0.03$ is used to verify that the numerical results are reproducible.

Several different nonlinear developments are observed to occur depending on the adverse shear–vorticity ratio Λ. For values of Λ between 0 and about 0.21, the strip of vorticity destablizes by rolling up into a string of vortices (see figure 3 for the case with $\Lambda = 0$, or no adverse shear, and compare with figure 4 which has $\Lambda = 0.2$). An active field of smaller strips or filaments surrounds and interacts with the largest vortices. Note in particular the prominent roll-up occurring in the case with $\Lambda = 0.2$ and the 'collision' between parts of the strips and the vortices in the case with $\Lambda = 0$.

Subsequently, the vortices would begin to 'pair' or merge (see Aref & Siggia 1980; Aref 1983; Ho & Heurre 1984; for theoretical considerations, see Pierrehumbert & Widnall 1981). This stage of the evolution is, however, not allowed by the constraint of periodicity imposed on these calculations. Even so, a straightforward extension of Lamb's (1932) analysis for the linear stability of a row of *point* vortices presented in Appendix A support the idea that adverse shear inhibits pairing, when $\Lambda \geqslant 0.1168\ldots$.

A second regime sets in for Λ between 0.21 and 0.45 (approximately). A calculation with $\Lambda = 0.23$ is illustrated in figure 5. The initial roll-up does not manage to break

Figure showing calculation	Duration of calculation	Phase error (degrees)
3	8.0	0.014
4	11.0	0.021
5	11.5	0.029
6	11.5	0.059
7	11.0	0.031
8	20.0	0.040
9	50.0	0.041 (0.020)
10	9.5	0.012
11	23.0	0.012
12	5.0	0.003

TABLE 1. Basic diagnostics of the calculations shown in this paper. Additional periodic calculations which were performed but not listed above include $\Lambda = 0.21, 0.22, 0.24, 0.45, 0.64, 0.7$ and 0.8. The phase error (ϵ_C) roughly measures the accumulative error in the lack of conservation of basic integral constants (e.g. circulation and angular momentum). See Dritschel (1988*a*, §4) for a discussion of errors and for the definition of ϵ_C in the non-periodic planar case. For the definition in the periodic case, see Appendix C of Dritschel (1988*c*). The two entries for figure 9 above correspond to calculations with $\mu = 0.04$ and 0.03 respectively.

the strip into a row of vortices; rather, the shear overcomes the roll-up and begins to extend each enlarged region of vorticity. Eventually, roll-up does occur on a much smaller scale at the places where the strip folds back on itself. At a still larger shear, $\Lambda = 0.33$, figure 6, even these secondary roll-ups are inhibited. In this regime, there is a competition between the formation of vortices by the instability mechanism of the strip and the destruction of vortices by the shear. While the strip is linearly unstable, it cannot manifest this instability by forming a row of vortices because the shear is too great. The fact that excessive shear precludes the formation of vortices is evidently related to the fact that an *isolated* (elliptical) vortex in pure shear will be extended indefinitely if $\Lambda > 3-2\sqrt{2} = 0.17157\ldots$ (Moore & Saffman 1971; Kida 1981). For a period array of vortices, the calculations suggest that the adverse shear must be slightly larger, say $\Lambda = 0.21$, to initiate extension. Note, however, that the extending strips will not continue to extend, apparently passively, as in figure 6, for they become susceptible to further strip instabilities once sufficiently aligned with the shear. On the basis of the results presented in Appendix B, it is likely that each thin strip in figure 6 will eventually fracture into a band of strips (one strip with many bends is intended) just as the primary strip in figure 6 did. Each of these secondary strips may likewise fracture again, and so on.

A third regime is observed for intermediate values of the adverse shear, $0.45 < \Lambda < 0.65$. A calculation with $\Lambda = 0.5$ is shown in figure 7. Vortices begin to form, are extended, and then torn into two. At the end of the calculation, the vortices begin to pair again, but not with the vortices they originally separated from. A great deal more is happening with the thinner strips of vorticity surrounding the vortices, including brief roll-ups. Near the high-shear end of the regime, $\Lambda = 0.6$, figure 8, the evolution is less disruptive. The strip is no longer extended as a whole, rather filaments are shed from the crests of the waves on the two interfaces. The waves then diminish in amplitude, but 'filamentation' continues to occur, with filaments on both interfaces being shed repetitively to the inner and outer sides of the strip (cf. figures 5 and 6 of Dritschel 1988*c*). Toward the end of the calculation, a large-scale wave pattern re-emerges and temporarily quells filamentation. The resemblance of this

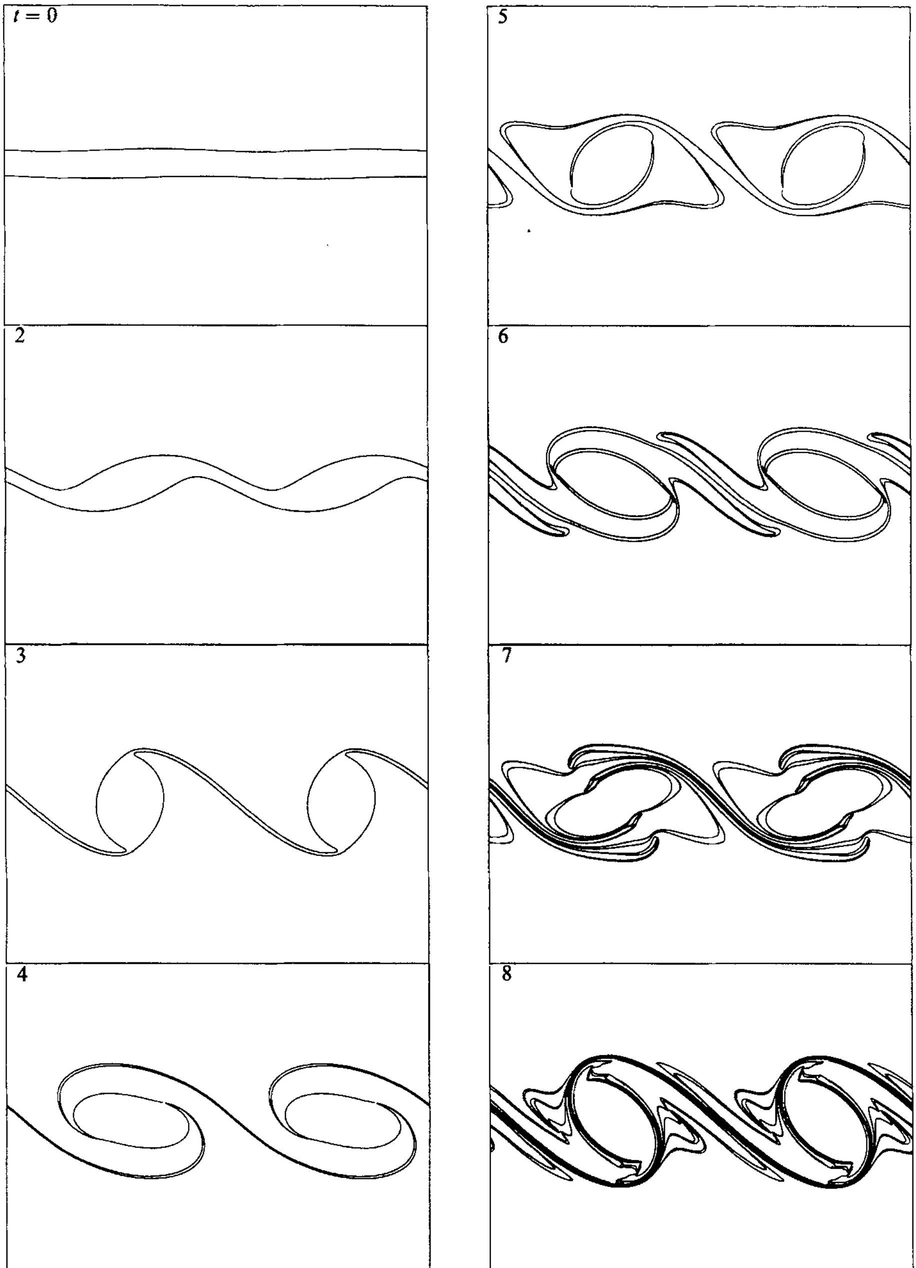


FIGURE 3. A contour-surgery calculation in periodic geometry of the instability of a strip of vorticity ($\Lambda = 0$). Two periods of the flow are shown, but only one was calculated. Time advances downwards and to the right. See table 1 for further details.

large-scale wave to that occurring earlier in the evolution suggests that waves will continue to decay, grow, and break, causing the interface (understood in an x-averaged sense) to spread further and further into the surrounding fluid.

In the fourth and final regime, $0.64 \leqslant \Lambda < 1$, no filament shedding is observed (see figure 9) through the duration of the calculations ($t = 50$). Initially, the disturbance

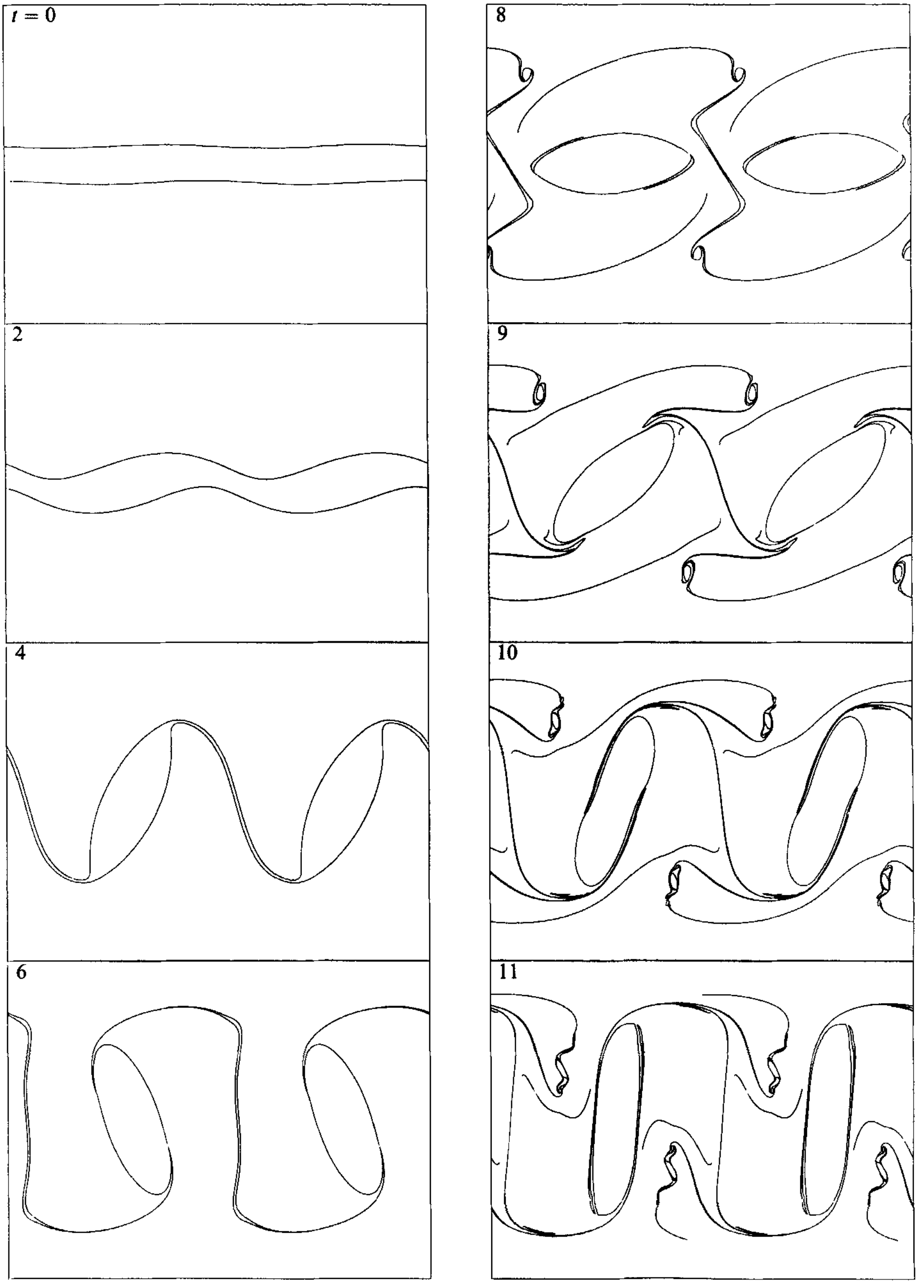


FIGURE 4. As in figure 3 but with $\Lambda = 0.2$.

amplifies until a trochoid-like wave pattern is established on the two interfaces, then, surprisingly, the waves decay until an almost unperturbed state is reached. There is then a long lull until the trochoid-like wave pattern reappears, with the same amplitude, and again the waves decay. This was such an unexpected result – particularly with regard to the absence of any wave breaking – that the calculation was repeated at higher resolution. The two calculations are superimposed at the time when the wave reaches maximum amplitude for the second time, at $t = 45$, showing the degree to which the calculation is reproducible.

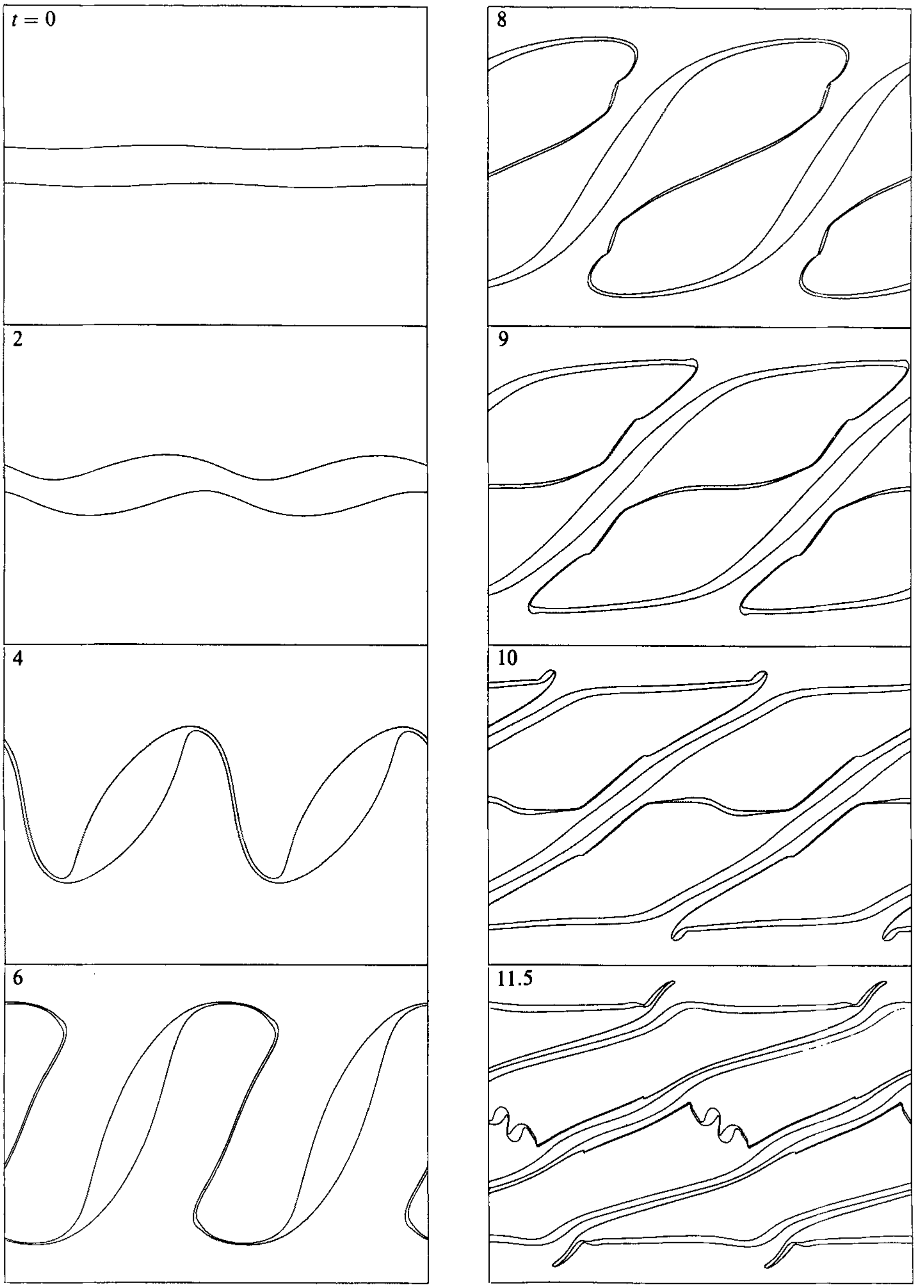


FIGURE 5. As in figure 3 but with $\Lambda = 0.23$.

Calculations with Λ closer to unity show less amplification – the maximum wave slope is less and the peak curvature is less. This is not to say that the evolution is exactly time-periodic; in fact, if it were possible to calculate the evolution with much higher resolution for much longer in time, one might well see filamentation occurring on a scale small compared with the wavelength (see Dritschel 1988*c* for support of this conjecture).

We next consider a single calculation in which the vorticity within the strip is not uniform. In this calculation, three distinct levels of vorticity are used, $\frac{1}{3}\omega$, $\frac{2}{3}\omega$, and ω,

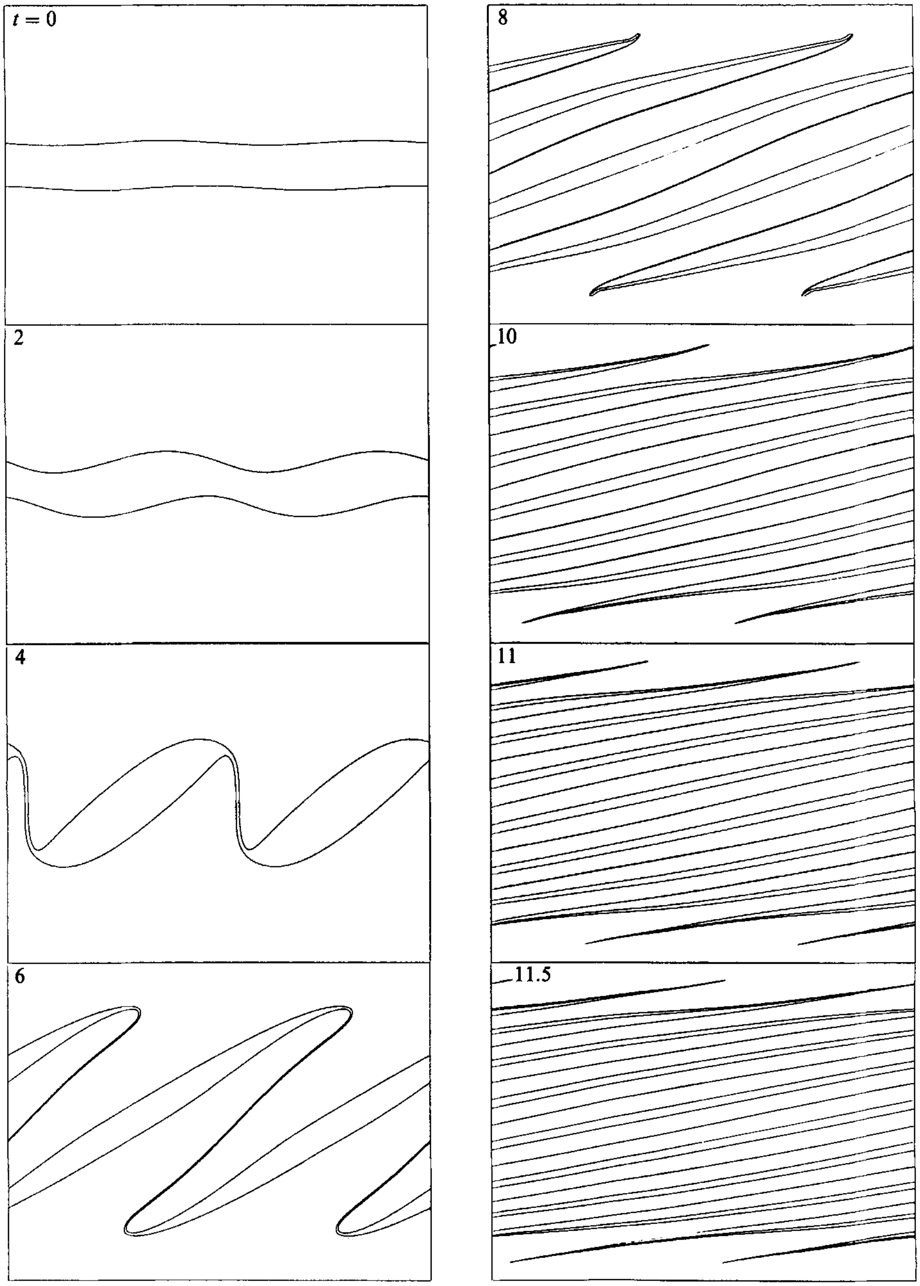


FIGURE 6. As in figure 3 but with $\Lambda = 0.33$.

where $\omega = 2\pi$ is the vorticity at the centre of the strip. The y-positions of the contours of vorticity discontinuity, in equilibrium, are taken to be $y_{\pm j} = \pm\frac{1}{2}\Delta(\frac{1}{3}j)^{\frac{1}{3}}$, $j = 1, 2, 3$, and the disturbance shifts each y-position by the amount $0.1y_{\pm j}$. The disturbance and strip width are chosen to correspond with the most unstable linear eigenmode for the *uniform* strip when $\Lambda = 0.2$. Figure 10(*a*) depicts the evolution. The dramatic difference between this and the uniform-vorticity case (cf. figure 4) is the 'stripping' of the weakest levels of vorticity from the forming vortices. By the

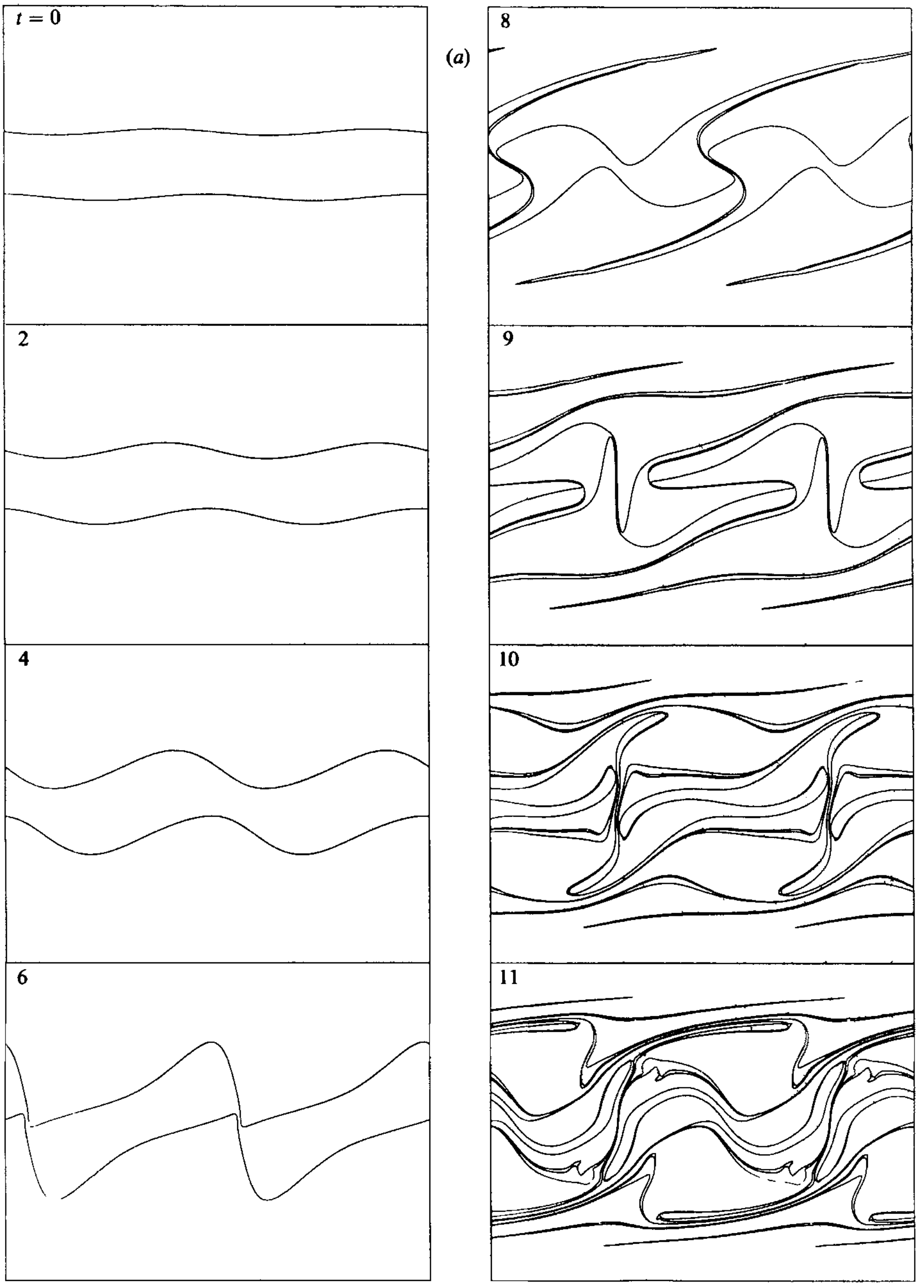


FIGURE 7(a). For caption see next page.

end of the calculation, almost all of the vorticity in the lowest two levels of vorticity has been stripped away into the fluid on either side of the strip. The fact that large-scale shear or strain efficiently removes weak vorticity and leaves vortices with gradients at their outer edge many orders of magnitude greater than in the initial state has been demonstrated in the case of isolated vortices (see Legras & Dritschel 1989 and §7 below). Essentially the same mechanism, it is argued, intensifies vorticity gradients in destabilizing strips.

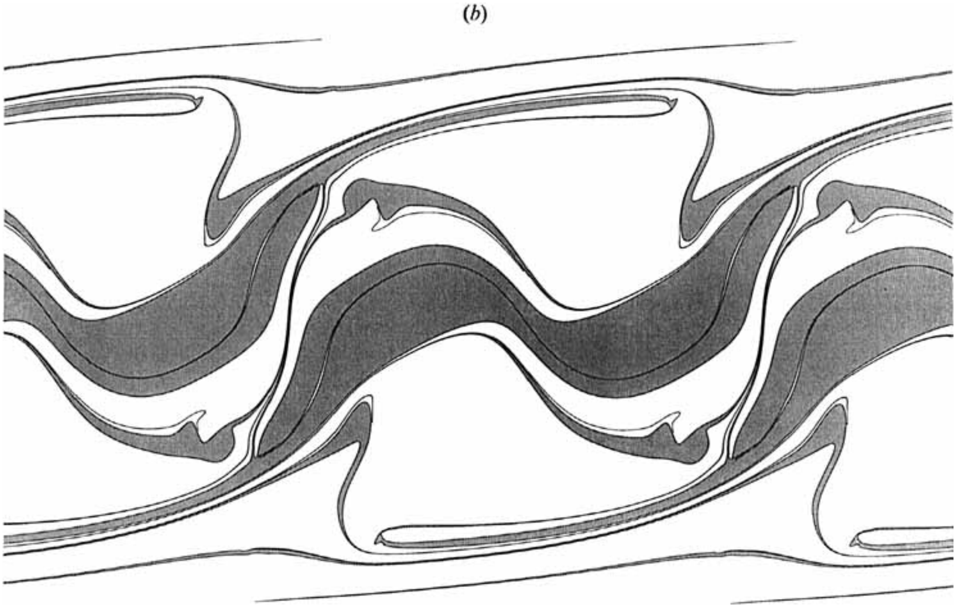


FIGURE 7. (a) As in figure 3 but with $\Lambda = 0.5$. (b) An enlarged view of the flow at $t = 11$.

Finally, we consider a circular strip of vorticity. Of particular interest is the case when a circular strip just encloses a central, finite vortex, in view of the instability uncovered in §4. Initially, the central vortex is circular (apart from numerical noise) with radius $R = 1$ and vorticity ω_0 equal to that in the surrounding strip, i.e. $\omega_0 = \omega = 2\pi$, and the strip lies between the radii $a = 1.08$ and $b = 1.10$. The evolution is illustrated in figure 11. The time difference $\Delta t = 2$ equals the rotation period, in equilibrium, of fluid particles along the edge of the central vortex. By ten such rotation periods, the character of the instability discloses itself. Not only does the strip begin to 'roll-up', in the sense opposite to that which would occur in the absence of a central vortex, but steep waves also grow from the boundary of the vortex. These waves subsequently tilt over and exude filaments of vorticity which then elongate in the differential rotation. Thereby new strips are born, and the flow continues to evolve with rapidly growing complexity. Despite the erosion of the central vortex by 'filamentation', this instability can never change the gross circular nature of the flow (Dritschel 1988*d*, §5).

7. Discussion

Thin strips being wound around intense vortices often behave quasi-passively because of the nature of the flow field in the immediate vicinity of the vortices. The strong differential rotation associated with the flow field at first thins strips of vorticity and aligns them with the flow. As the strips align with the flow, the rate of stretching continually diminishes, and eventually this stretching is insufficient to prevent instabilities on the basis of strain alone (Dritschel *et al.* 1989). But the effect of differential rotation continues to exert its influence, in the form of adverse shear, by opposing a strip's own self-induced angular velocity shear. Linear stability is assured if the adverse shear at least cancels the strips self-induced shear; nonlinear

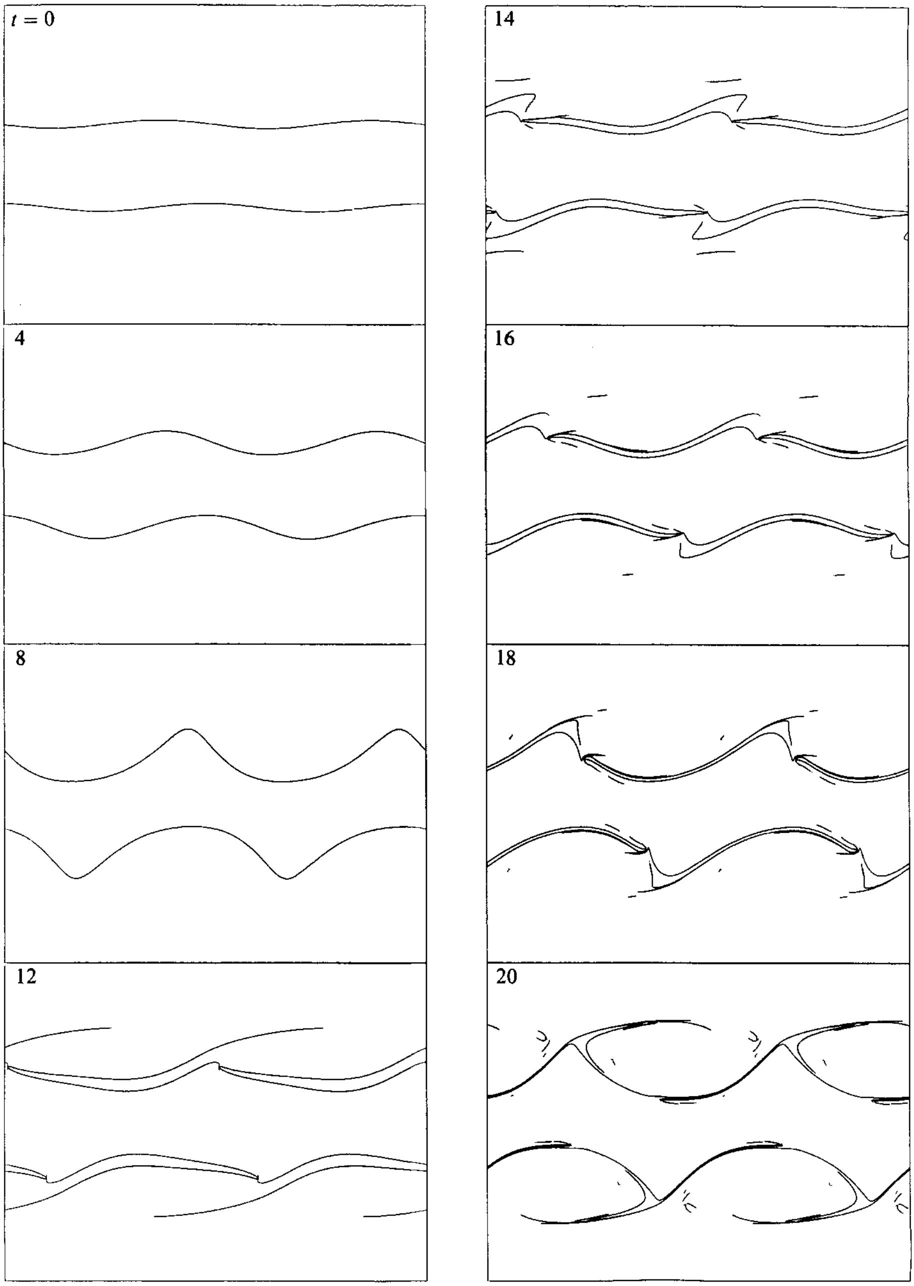


FIGURE 8. As in figure 3 but with $\Lambda = 0.6$.

stability appears to require significantly less adverse shear. On the basis of the nonlinear results, it appears that a thin strip with (peak) vorticity ω at a radial distance r from the centre of an intense vortex will remain strip-like so long as the circulation-like quantity $\pi r^2 \omega$ does not exceed 1.5 times the circulation of the vortex.

However, if a strip is too close to an intense vortex, an instability takes place in which parts of the strip are pulled toward the vortex while the vortex itself develops steep boundary waves. These waves then proceed to generate filaments repeatedly

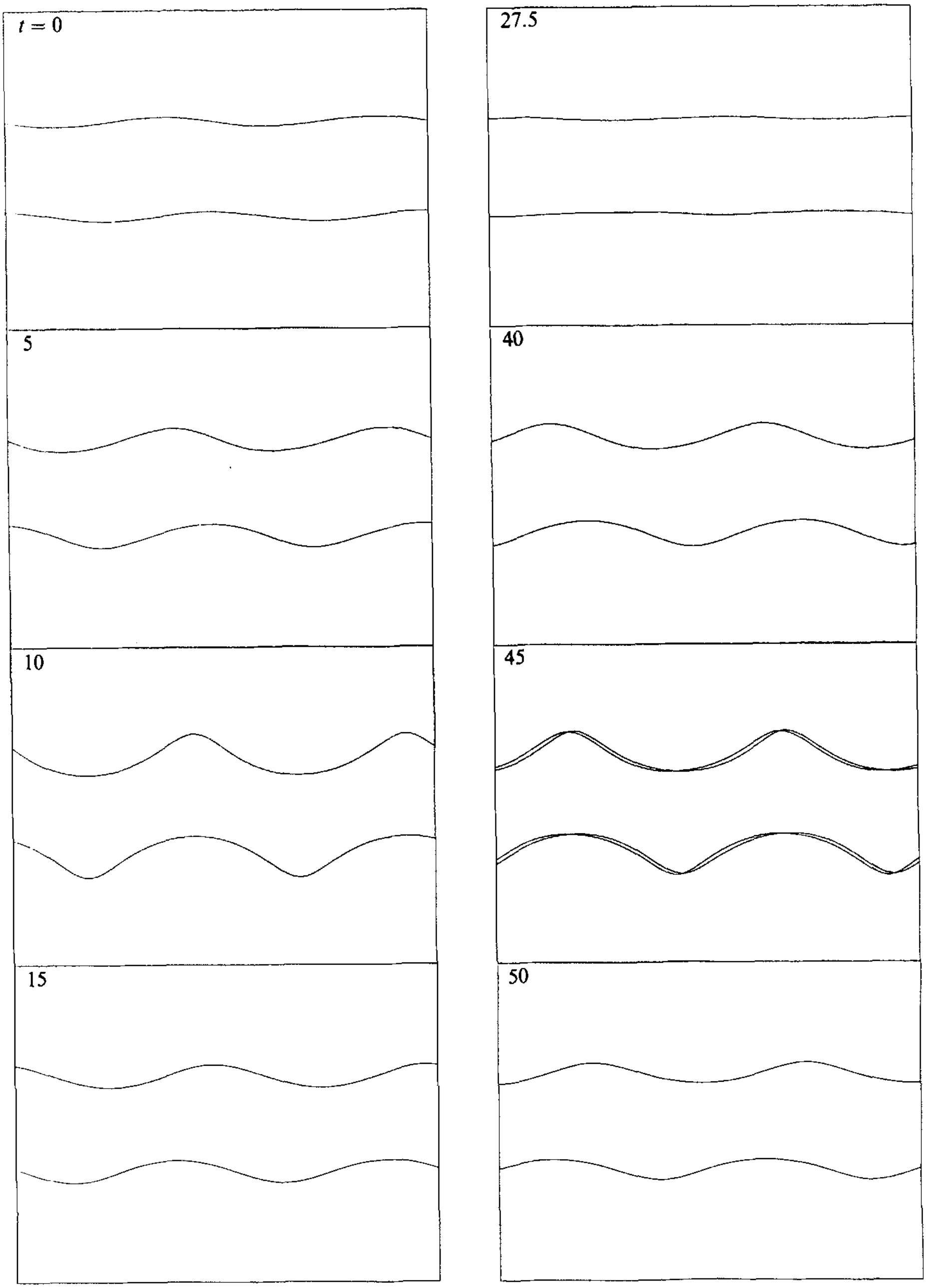


FIGURE 9. As in figure 3 but with $\Lambda = 0.65$. The second set of contours in the plot for $t = 45$ were obtained at higher resolution ($\mu = 0.03$ instead of $\mu = 0.04$). The difference between the two calculations is largely a phase error.

(Dritschel 1988*c*) thereby continually adding new strips to the flow at the edge of the vortex. The new strips themselves may eventually go unstable only to leave the edge of the vortex with yet more strips. It is thought that this complex behaviour reduces vorticity gradients, in a coarse-grained sense, at the edge of the vortex and makes the vortex less sensitive to small disturbances such as nearby strips (Dritschel 1988*c*, §6, 1988*d*).

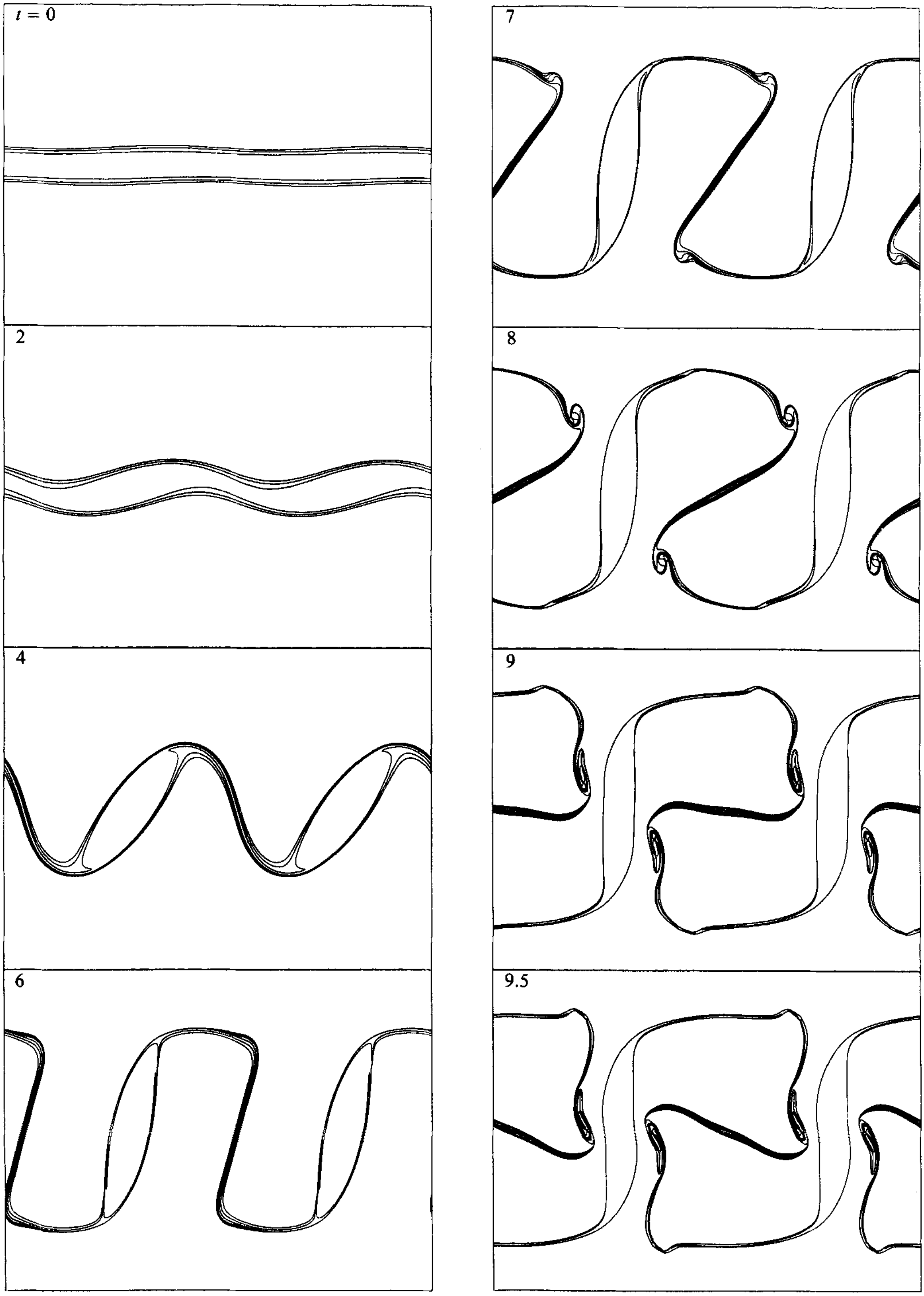


FIGURE 10. As in figure 4 but with three levels of vorticity.

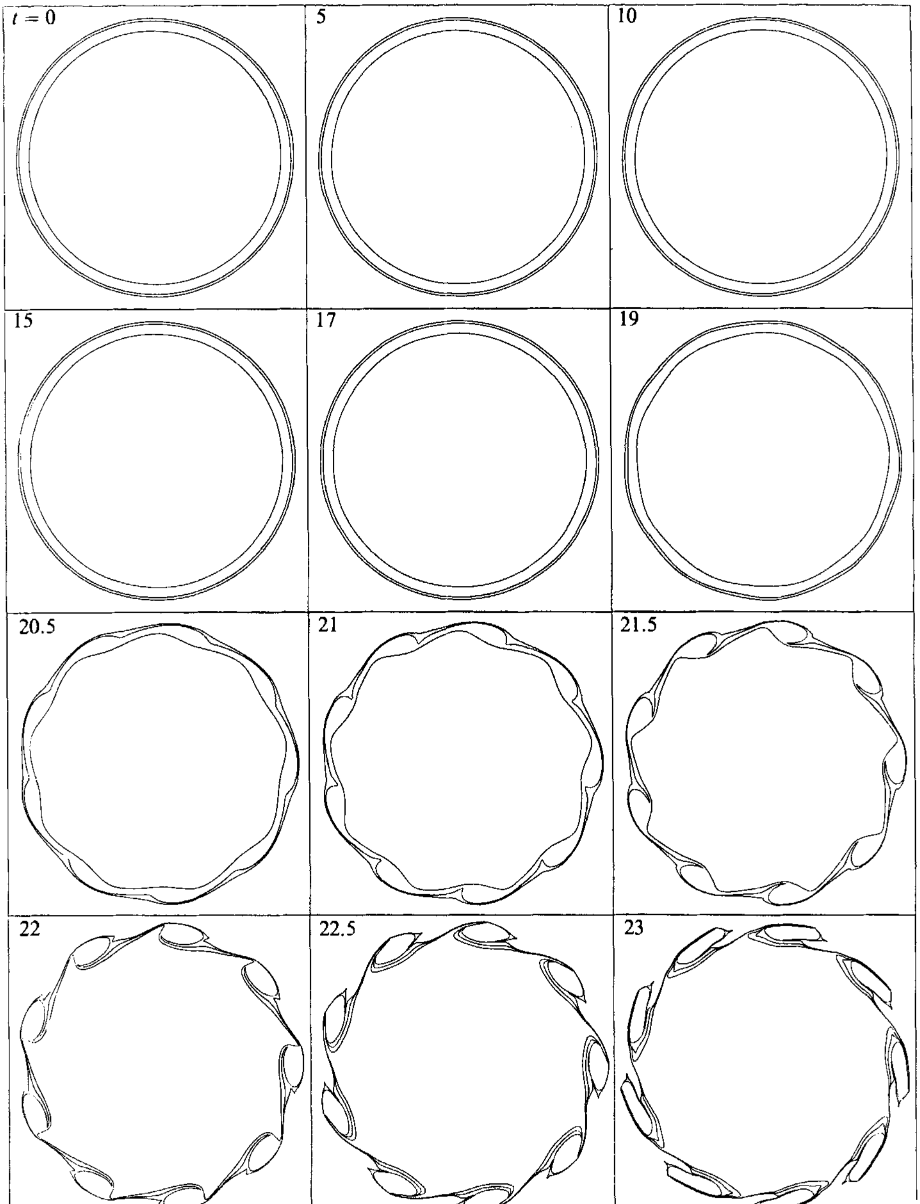


FIGURE 11. A calculation of the nonlinear stability and evolution of a thin strip of vorticity surrounding a central vortex of the same uniform vorticity. Time advances to the right and downwards. The parameters $\mu = 0.02$, $\delta = 0.00005$, and $\Delta t = 0.05$. See table 1 for error information.

So far, no attention has been given to the effect of distant vortices. The large-scale strain and shear arising from distant vortices need not be great for there to be a significant effect on the flow around a particular vortex. It has been known for some time that a uniform strain of only 15% of the vorticity applied to a *uniform* vortex will cause the entire vortex to elongate irreversibly (Moore & Saffman 1971; Kida 1981; Neu 1984). As noted in the previous section, adverse shear exceeding about 17% of the vorticity has a similar effect. And so it is perhaps not surprising that somewhat weaker strain or shear can tear away (strip) practically all of the weakest levels of vorticity from the outer edges of any vortex having a continuous distribution of vorticity (Legras & Dritschel 1989). This 'stripping' can leave the surviving part of the vortex with gradients of vorticity four to six orders of magnitude greater at its edge than it had initially, and after a time of the order of only one rotation of the vortex! Of course, the strain rate due to distant vortices is not constant in turbulent flows, and vortices experience a variety of external flow conditions. Nevertheless, weak large-scale strain and shear offer an efficient way to remove bundles of strips encircling intense vortices and provides a very effective way to intensify vorticity gradients (cf. Melander *et al.* 1987*a*).

The strips pulled away from vortices can occasionally become unstable when they reach regions of the flow with favourable strain and shear, to produce the familiar, but in practice seldom noticeable shear instability associated with an isolated strip of vorticity. Some of the recent very high-resolution numerical calculations of two-dimensional turbulence have indeed detected this instability (Juckes & McIntyre 1987, figure 4*e*; B. Legras, personal communication 1987), but it appears to be rare. Usually, strips do not have sufficient time to behave as if they were in isolation. The local, large-scale strain field typically changes sufficiently rapidly to keep a particular mode of instability from growing very much (cf. Dhanak 1981; Dritschel *et al.* 1989).

Shear instability is most likely to occur with strips whose vorticity values are comparable with the peak vorticity values in the flow and particularly to strips being wound around vortices with opposite-signed vorticity. In this latter case, the shear due to the vortex is no longer adverse, but in the same sense as that in the strip, and this causes the strip to be even more unstable than when the strip is in isolation (see figure 12). But the fact remains that shear instability has been rarely observed in the high-resolution physical and numerical experiments cited in §1. The thin bands of vorticity that do get wrapped around vortices with opposite-signed vorticity probably become extremely thin as a result of the accumulative stretching experienced during their birth from a vortex of the same-signed vorticity, their removal from the parent vortex by the strain field of distant vortices, and their eventual capture by a vortex with opposite-signed vorticity. By the time a strip is ready to undergo its own instability, if the strip has not already been removed again by the strain field of distant vortices, the strip would probably be overcome by viscous effects (or be lost in a numerical model's truncation).

This and the complementary study (Dritschel *et al.* 1989) on the effects of large-scale shear and strain on the behaviour of thin strips or filaments of vorticity in conjunction with results on gradient intensification through vortex 'stripping' (Legras & Dritschel 1989) and the 'filamentation' of steep vorticity gradients (Dritschel 1988*c*), lend considerable support to one part of the classic Batchelor–Kraichnan two-dimensional turbulence scenario (Batchelor 1969; Kraichnan 1967), namely the idea that small scales behave quasi-passively on the whole. Together with the more recently recognized role of concentrated, sparsely spaced vortices (Fornberg 1977; Basdevant *et al.* 1981; McWilliams 1984 among others), a picture is emerging wherein the dynamics of the concentrated vortices, their collisions, their deformation by the interaction with distant vortices, and their evolving internal structure, comprises the most visible and important element in a turbulent flow. Such a picture is likewise suggested by the very high-resolution calculations of Babiano *et al.* (1987), Benzi *et al.* (1987), and Legras *et al.* (1988) and envisioned by Melander *et al.* (1987*a*, *b*, *c*) who have examined the dynamics of one or two vortices in isolation.

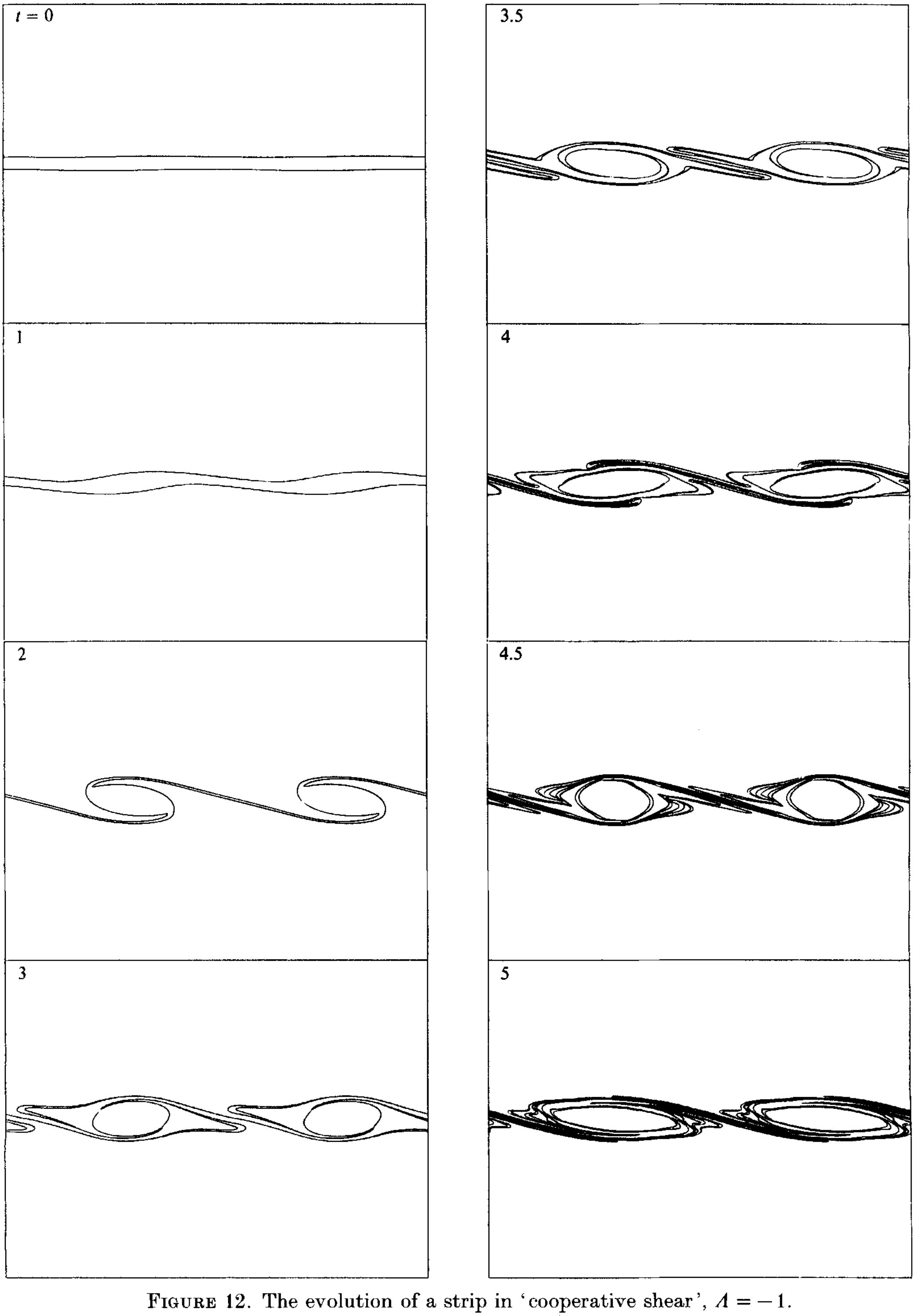


FIGURE 12. The evolution of a strip in 'cooperative shear', $\Lambda = -1$.

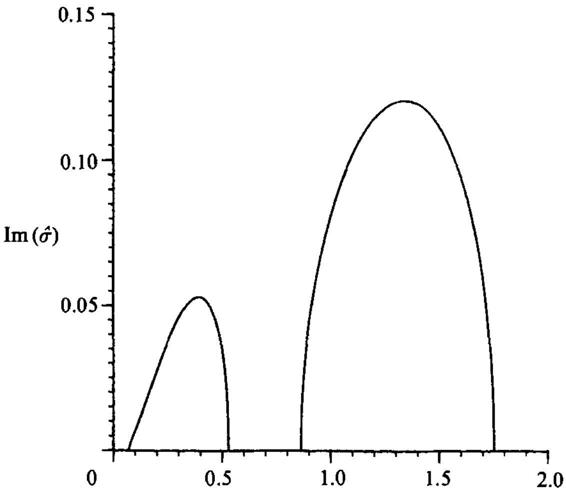


FIGURE 13. The dimensionless growth rate versus dimensionless wavenumber $k\Delta$ for two strips separated by a distance $D = 5$ from centre to centre in adverse shear of $\Lambda = 0.33$. In the right lobe of instability, the structure of an eigenmode disturbance on either strip is nearly identical to that on a single strip in complete isolation. However, in the left mode of instability, the two strips interact strongly.

Special thanks are due to M. E. McIntyre and B. Legras for their keen awareness of the small-scale structures in fluid dynamics and for their remarks concerning this paper. I also wish to thank S. P. Cooper for providing figure 7(*b*). The computations were performed on the Cray X-MP/48 at the Rutherford Appleton Laboratory under the UK Universities Global Atmospheric Modelling Project, with the support from the Natural Environmental Research Council.

Appendix A. Stabilizing a row of vortices with adverse shear

It is shown that the vortices that roll-up from the strip instability when $\Lambda < 0.21$ do not subsequently begin pairing when $\Lambda > 0.1168\ldots$, under the assumptions made below.

Suppose $\Lambda < 0.21$ and the most unstable eigenmode overwhelms all others to produce a row of vortices like those shown in figures 3 and 4. Assume that all of the vorticity in each period of the strip (of wavelength $a = 2\pi/k_m$and initial width Δ, where the product $k_m \Delta$ is a function of Λ as given in figure 2*b*) ends up in a single vortex (in each period) and that the resultant row of vortices, insofar as the following linear stability analysis is concerned, can be approximated by a row of point vortices, each point vortex having the same circulation $\kappa = \omega\Delta a$ as each finite vortex.

The linear stability analysis is a minor modification of that presented by Lamb (1932, pp. 225–226). Referring to his analysis, one simply adds the term $\omega\Lambda y_0$ to the right-hand sides of his equations involving $\mathrm{d}x_0/\mathrm{d}t$ (equations (7) and (9)), and $\omega\Lambda\beta$ in a similar way to $\mathrm{d}\alpha/\mathrm{d}t$ (in equation (11)), this latter modification showing that stability results if $\omega\Lambda$ exceeds the maximum of λ, or

$$\Lambda \geqslant \frac{\pi\kappa}{4a^2\omega} = \tfrac{1}{8}k_m\Delta.$$

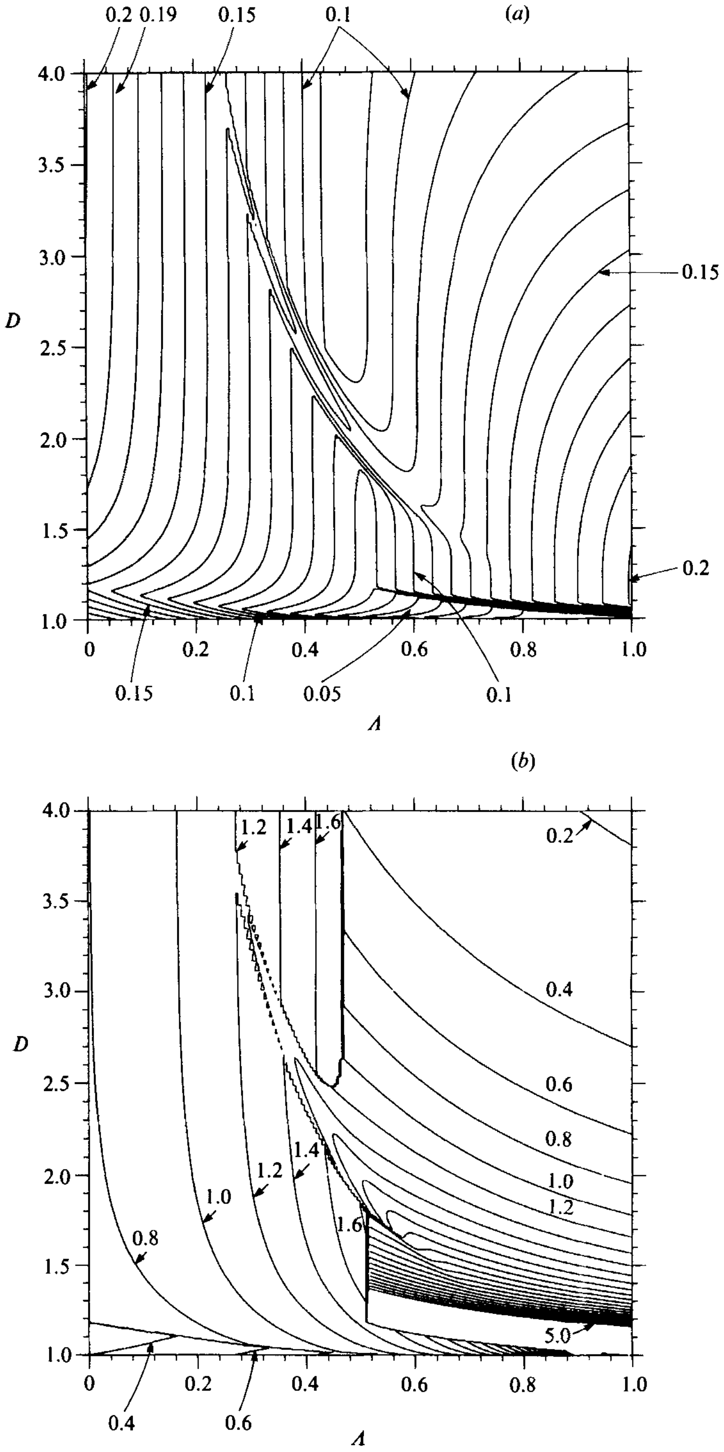


FIGURE 14. (*a*) The maximum dimensionless growth rate as a function of Λ and D for two strips. The wavenumbers $k\Delta > 10$ are excluded. Their inclusion would cause the vertical contours in the lower right-hand part of the picture to intersect the lower horizontal axis (these modes are characterized, principally, by the excitement of the thin gap between the two strips in the limit $D \rightarrow 1$). Note that the sufficient condition for stability reviewed in §5 is never satisfied for this flow. (*b*) The dimensionless wavenumber of the most unstable eigenmode.

Since $k_m \Delta$ is itself a function of Λ (from figure 2*b*), one can solve for the value or values of Λ when $\Lambda = \frac{1}{8}k_m \Delta$. One finds two roots, the smallest at $\Lambda = 0.11680634\ldots$ and the largest at $\Lambda = 0.85388480\ldots$. Between these two values of Λ, there is neutral stability; hence, the vortices that form as a manifestation of the most vigorous linear instability of a vortex strip do not subsequently begin to pair if $\Lambda > 0.1168\ldots$. Since strips with values of Λ exceeding the larger root, $\Lambda = 0.8538\ldots$, show little disruption as a result of their linear instability (certainly, they do not break up), this larger root has no physical significance.

Appendix B. The stability of two parallel strips in adverse shear

The question of whether the secondary strips in figure 6 eventually fracture into tertiary strips, and so on, is examined here. We consider the simplest possible problem of two parallel strips, each of vorticity ω and width Δ separated by a distance $D\Delta$ from centre to centre. We also suppose there is a background adverse shear present, with shear $\Lambda\omega$. In figure 6, $\Lambda = 0.33$, and it is estimated that $D = 5$.

The dispersion relation for the eigenfrequency σ is straightforwardly obtained by a generalization of (9) along with a bit of algebra. With $\hat{\sigma} = \sigma/\omega$, $\kappa = k\Delta$, $\alpha = 1-\kappa\Lambda(D-1)$, $\beta = 2\kappa-1-\kappa\Lambda(D+1)$, and $\gamma = e^{-2\kappa}$, stability is determined from

$$16\hat{\sigma}^4-(\alpha^2+\beta^2-2\gamma-\gamma^{D-1}+2\gamma^D-\gamma^{D+1})\,4\hat{\sigma}^2 +(\alpha\beta+\gamma)^2-\gamma^{D-1}\beta^2+2\gamma^D\beta(\alpha-2)-\gamma^{D+1}(\alpha-2)^2 = 0.$$

Figure 13 shows $\mathrm{Im}\,(\hat{\sigma})$ versus $\kappa = k\Delta$ for the case corresonding to figure 6. The second strip gives rise to a second mode of instability having longer wavelengths. For this mode, the two strips are strongly coupled. The more unstable mode at shorter wavelengths corresponds directly to the single-strip mode, and the two strips behave almost independently. Since this latter mode dominates, it is probable that the subsequent evolution in figure 6 will exhibit further strip fracturing. Although this argument depends upon the results for only two strips, it should be qualitatively correct for any number, as long as the distance between the strips is much greater than the width of the strips.

More generally, figure 14 displays maps of the growth rate maximized over wavenumber and the wavenumber of maximum instability in the domain $0 \leqslant \Lambda \leqslant 1, 1 \leqslant D \leqslant 4$. For large D, the results asymptote to those corresponding to a single strip, and for $D = 1$, the results apply to a single strip of width 2Δ. The various lines of discontinuity evident in the map for $k\Delta$ reflect crossovers from one type of mode to another.

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